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Nuclear pairing and Coriolis effects in proton emitters

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Abstract. We introduce a Hartree-Fock-Bogoliubov mean-field approach to treat the problem of proton emission from a deformed nucleus. By substituting a rigid rotor in a particle-rotor model with a mean field, we obtain a better description of experimental data in ^{141}Ho. The approach also elucidates the softening of kinematic coupling between particle and collective rotation, the Coriolis attenuation problem.

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Proton emission is a weak single-particle (s.p.) process with widths about 20 orders of magnitude smaller than the usual MeV scale of other nuclear interactions. This makes observation of proton radioactivity an ideal and powerful tool for non-invasive probing of the single-proton in-medium dynamics. Recent studies have already explored numerous nuclear mean-field properties of proton emitters including deformations, vibrations [1] rotations [2], pairing and other many-body correlations [3,4].

In this work, using proton emission from deformed nuclei, we concentrate on an old problem known as *Coriolis attenuation problem* [5] in the particle-rotor model (PRM). Recent studies of proton decay [2,4] highlight the same lack of kinematic coupling between the particle and the deformed rotor as was inferred decades ago from observations of the energy spectra of odd-A nuclei [5,6]. The second purpose of this work is to gain an understanding of and to develop a better theoretical technique to describe particle motion in the deformed mean-field. Here the notion of a core as a rigid rotor is inadequate and, as emphasized in numerous works [5,7,8], the residual two-body interaction and collective modes are important parts of the dynamics.

We consider an axially-symmetric deformed proton emitter and assume that the total Hamiltonian is composed of a collective $H_{\rm coll} = \mathbf{R}_\perp^2/2\mathcal{L}$ and intrinsic parts

$$H_{\rm intr} = \sum_{\Omega} \epsilon_\Omega\, a^\dagger_\Omega a_\Omega \, - \frac{1}{4} \sum_{\Omega\Omega'} G_{\Omega\Omega'} a^\dagger_{\tilde\Omega} a^\dagger_\Omega \, a_{\Omega'} a_{\tilde\Omega'} \, . \quad (1)$$

Here $\mathbf{R}$ denotes the rotor angular momentum, involving only the part perpendicular ($\perp$) to the symmetry axis, and $a^\dagger_\Omega$ and a_Ω stand for s.p. creation and annihilation operators of state $|\Omega)$ in the deformed body-fixed mean-field potential. Nuclear pairing involves body-fixed time-reversal s.p. states $|\Omega)$ and $|\tilde\Omega)$ and describes the residual two-body interaction. In contrast to the usual PRM this model assumes some odd number of valence particles. In the limit where the valence space covers the entire nucleus the collective rotor variables become redundant.

Kinematic coupling between the intrinsic system and collective rotor occurs due to conservation of total angular momentum $\mathbf{I} = \mathbf{R} + \mathbf{j}$, where $\mathbf{j}$ is the angular momentum of the valence particles. Components of this operator can be expressed in the a intrinsic body-fixed basis as

$$j_3 = \sum_\Omega \Omega_\Omega a^\dagger_\Omega a_\Omega, \quad j_+ = \sum_{\Omega\Omega'} j_{\Omega\Omega'}\, a^\dagger_\Omega a_{\Omega'}, \quad (2)$$

similarly for $j_- = j_+^\dagger$. The coefficients $j_{\Omega\Omega'} = (\Omega|j_+|\Omega')$ are obtained using expansion of states $|\Omega)$ in spherical basis. Excluding a trivial rotational part from the total Hamiltonian $H = \mathbf{I}^2/(2\mathcal{L}) + H'$, we obtain

$$H' = \frac{1}{2\mathcal{L}}(\mathbf{j}^2 - 2j_3^2) - \frac{1}{2\mathcal{L}}(j_+ I_- + j_- I_+) + H_{\rm intr}, \quad (3)$$

which is to be solved via many-body techniques using basis states formed as products of Wigner $D^I_{MK}(\omega)$-functions of collective angles ω, and any complete set of many-body intrinsic states such as Slater determinants.

Here we implement a Hartree-Fock-Bogoliubov (HFB) approach that allows one to determine a s.p. mean-field, which is a combination of the rotor degrees of freedom and even-particle valence system, and absorbs in the best way kinematic couplings and residual nucleon-nucleon correlations. By making a Bogoliubov transformation to quasiparticles $\alpha_i = \sum_\Omega \left(u^i_\Omega a_\Omega + v^i_\Omega a^\dagger_\Omega\right)$ and with the require-

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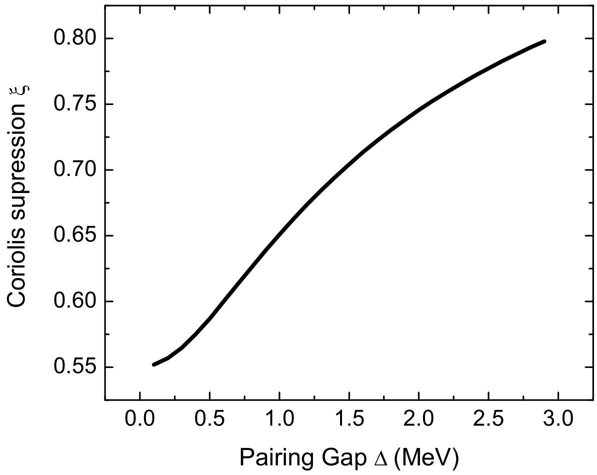

Fig. 1. The average Coriolis suppression factor as a function of the pairing gap in ^{141}Ho.

ment that the elementary quasiparticle excitations are stationary we obtain the usual HFB equations

$$u^i_\Omega e_i + \Delta_\Omega {v^i_\Omega}^* = \sum_{\Omega'} \varepsilon_{\Omega\Omega'} u^i_{\Omega'} \,,$$

$$v^i_\Omega e_i + \Delta_\Omega {u^i_\Omega}^* = -\sum_{\Omega'} \varepsilon_{\Omega\Omega'} v^i_{\Omega'} \,. \tag{4}$$

Here in full analogy to PRM the diagonal part of the s.p. potential is given by the usual s.p. energy corrected with the recoil term and decoupling factor ΔE [5]

$$\varepsilon_{\Omega\Omega} = \epsilon_\Omega + \frac{1}{2\mathcal{L}} \left((\Omega|\mathbf{j}^2|\Omega) - 2\Omega + \delta_{\Omega,1/2}\,\Delta E \right) . \tag{5}$$

The off-diagonal term in eq. (4) violates deformation alignment, the K-symmetry, which manifests itself through non-vanishing average mean-field expectations $\langle j_+ \rangle = \langle j_- \rangle = \langle j \rangle$ while $\langle j_3 \rangle = 0$. This average mean-field value enters the off-diagonal s.p. potential

$$\varepsilon_{\Omega+1,\Omega} = -\frac{1}{2\mathcal{L}} \left[\sqrt{(I-\Omega)(I+\Omega+1)} - \langle j \rangle \right] j_{\Omega+1\,\Omega}, \tag{6}$$

and is to be determined in a self-consistent solution

$$\langle j \rangle = 2 \sum_{i,\,\Omega>0} j_{\Omega+1,\Omega}\; v^i_{\Omega+1} v^i_\Omega \,. \tag{7}$$

This is analogous to non-conservation of particle number N, a common situation in the HFB approach. Particle number is restored on average via the introduction of a chemical potential $H' \to H' - \mu N$, so that the pairing gap and chemical potential in eq. (4) are self-consistently determined

$$\Delta_\Omega = -\frac{1}{2} \sum_{\Omega'} G_{\Omega\Omega'} \sum_i u^i_{\Omega'} v^i_{\Omega'}, \;\; N = 2 \sum_{\Omega>0} \sum_i {v^i_\Omega}^* v^i_\Omega . \tag{8}$$

The term $\langle j \rangle$ in eq. (6) is due to HFB linearization of the recoil operator $\mathbf{j}^2 \sim \langle j \rangle (j_+ + j_-)/2 + \Omega^2$ which,

Table 1. Comparison of different theoretical results and experimental data for the case of ^{141}Ho proton emission.

	Γ_0 $(\times 10^{-20}$ MeV)		$\Gamma_2/\Gamma_0(\%)$	
	PRM	RHBF	PRM	RHFB
Adiabatic	15.0	15.0	0.73	0.73
Coriolis	1.4	5.9	1.8	1.2
Coriolis + pairing	1.7	7.0	1.7	0.3
Experiment	10.9 ± 1.0		0.71 ± 0.15	

besides acting on an odd particle, also perturbs an even-particle mean-field, thus producing a suppression of the Coriolis mixing. The Coriolis interaction takes the form $-(\mathbf{I} - \langle \mathbf{j} \rangle)_\perp \mathbf{j}/\mathcal{L}$ similar to the Routhian in the Cranking Model [5], and is suppressed. This is in contrast with the PRM, where by definition the rotor is rigid and $\langle j \rangle = 0$. The quantity $\xi = \left(1 - \langle j \rangle / \sqrt{I(I+1) - \langle \Omega^2 \rangle}\right)$ is the average suppression factor; for the case of ^{141}Ho (see below), it is shown as a function of pairing gap in fig. 1. The idea to phenomenologically substitute the spin of the rotor $\mathbf{R} = (\mathbf{I} - \mathbf{j})_\perp$ for the operator $\mathbf{I}$ in order to explain Coriolis attenuation was suggested in [9], and contributions from the $\mathbf{j}^2$ operator in the mean-field approach are discussed in [10]. Other contributions coming from non-rigidity of the core are also considered [5,8].

We apply this approach to the proton emitter ^{141}Ho where partial decay widths Γ_0 for decay to the 0^+ ground state and Γ_2 to the 2^+ first excited state in ^{140}Dy are known from experiment. The spectrum of ^{140}Dy is used to determine deformation and moment of inertia. The valence space is limited to a negative parity subspace coming from spherical $h_{11/2}$ orbital, but particle depletion due to pair excitation onto positive parity states is included. The decay amplitudes computed using appropriate deformed Woods-Saxon potential and expressed via normalization of the wave function [2] $A^\Omega_{lj}(k) = \phi^\Omega_{lj}(r)/G_{lj}(kr)\Big|_{r=\infty}$, where G_{lj} is the irregular Coulomb function. The decay width is given by [4] $\Gamma = \frac{k}{\mu} \frac{2(2R+1)}{2I+1} \left| \sum_{\Omega>0} C^{IK}_{jK,R0}\, u^i_\Omega A^\Omega_{lj} \right|^2$, where C is a Clebsch-Gordan coefficient and the u_Ω factors come from the solution of eq. (4). The results of this calculation, labeled as RHFB, are compared with PRM and experiment in table 1. The Coriolis attenuation problem is transparent; *e.g.*, for Γ_0 (first column), the unjustified theoretically adiabatic limit ($\mathcal{L} \to \infty$) overestimates experiment. When improving this by introduction of Coriolis mixing which is softened by pairing correlations the result extremely over-reduces Γ_0. The HFB calculation shown in table 1 is limited to a very small valence space, but gives a reasonable description, and most importantly, as a better founded approach clarifies the reason for weakened Coriolis coupling.

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